SENSITIVITY OF BED SHEAR STRESS ESTIMATED FROM VERTICAL VELOCITY PROFILES: THE PROBLEM OF SAMPLING RESOLUTION

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ABSTRACT

Bed shear stress in open channel flows is often estimated from the logarithmic vertical velocity profile. However, most measuring devices used in the field do not allow for flow velocity to be measured very close to the bed. The lack of near-bed measurements is a critical loss of information which may affect bed shear stress estimates. Detailed velocity profiles obtained from a field acoustic Doppler velocimeter over three different bed roughnesses clearly show that the inclusion of near-bed points is critical for the estimation of bed shear stress in a shallow river environment. Moreover, the results indicate that using the full flow depth instead of the bottom 20 per cent of the profile generates an underestimation of the shear stress when flow is uniform. © 1998 John Wiley & Sons, Ltd.

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INTRODUCTION

The vertical variation of velocity with elevation above the bed has often been used to estimate bed shear stress in alluvial channels (Bridge and Jarvis, 1976, 1977; Ashworth and Ferguson, 1986; Petit, 1990; Hassan and Reid, 1991; Robert *et al*., 1992). From the von Karman–Prandtl law of vertical velocity distribution, also known as the ‘law of the wall’, velocity can be related to the logarithm of height:

$$u = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \tag{1}$$

where u is the time-averaged streamwise velocity at elevation z above the bed, κ is von Karman’s constant (generally set to 0·4), u_* is the shear velocity and z_0 is the roughness height. The bed shear stress, τ_0, can be estimated from the shear velocity:

$$\tau_0 = \rho\ u_*^2 \tag{2}$$

where ρ is the water density.

Although more recent estimation of shear stress values has focused upon direct derivation of the Reynolds shear stresses from turbulence measurements (e.g. Heathershaw, 1979; Williams *et al*., 1989; Biron *et al*., 1993), use of the law of the wall allows measurement of average bed shear stress without recourse to sophisticated high frequency flow measurement instrumentation. This method has not been without problems, particularly in shallow water environments. In most field studies, the minimum height of velocity measurement above the bed is relatively large (>0·02 m) owing to the difficulty of measuring close to the bed (e.g. Ferguson and Ashworth, 1992; Robert *et al*., 1992). It may be difficult to obtain several velocity measurements in the portion of the flow where the law of the wall applies, i.e. the bottom 20 per cent of the flow depth (Bathurst, 1982; Nezu and

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Nakagawa, 1993). Many workers have taken a site-specific approach and the proportion of depth considered to be semi-logarithmic varies considerably from one study to another (e.g. 15 per cent in Bridge and Jarvis (1977, 1982); 20 per cent in Bridge and Jarvis (1976); over 50 per cent in Ferguson *et al*. (1989) and Ferguson and Ashworth (1992)). At present, the implications of the selection of measurement heights on the estimation of bed shear stress using a fitted logarithmic profile are unknown, although generalization will be difficult due to the site-specific nature of the problem.

The objective of this study is to simulate the uncertainty that arises from using the law of the wall in shallow streams, using a specific case-study to assess the extent to which estimates could be in error. More particularly, the effects on bed shear stress of varying (i) the range of flow depths from which observations are obtained, and (ii) the lowest elevation of a measurement point above the bed, are examined. These issues are investigated using a high vertical definition of velocity measurements taken in a stream with mixed sand and gravel bed.

METHODS

This research made use of a Sontek field Acoustic Doppler Velocimeter (ADV) which measures instantaneous flow velocity in three dimensions. The following were most important for this research: (i) that the measuring volume was small ($0{\cdot}125\,cm^3$); (ii) that it was situated 5 cm below the sensor head; and (iii) that the instrument sensed the distance between the bottom of the measuring volume and the bed surface to a high degree of accuracy (±mm). This allows precise determination of the position of each velocity measurement above the bed (cf. Bergeron and Abrahams, 1992), and acquisition of a high-resolution record of the vertical variation of velocity with elevation with minimal profile disturbance due to the presence of the probe. Data were sampled from three field locations of different roughness in shallow clear water rectilinear tributaries of the braided river flowing from the Haut Glacier d'Arolla (Switzerland). Sediment samples were collected at each reach and were later dry-sieved to obtain the grain size distribution. Discharge was low and the bed was not active in all three cases. Table I provides summary characteristics for the three locations.

At each location, the ADV was mounted on a specially designed wading rod which ensured that the vertical component of velocity was measured perpendicular to the horizontal. The sites were chosen such that cross-stream flow components were minimal, and the instrument was oriented in the streamwise direction. The instrument was moved up the wading rod from the minimum elevation of the measuring volume given in Table I to the maximum permissible without the probe head becoming exposed at the water surface. At each elevation, a 2 min series was collected at 25 Hz frequency. All measurements were obtained during periods of constant discharge.

The results were analysed to determine the time-averaged downstream velocity (u) at each elevation (z). To determine the bed shear stress using the law of the wall, u was regressed against $\ln z$. The three data sets were subsampled in two forms. First, the effect of using the full flow depth versus the near-bed portion of the velocity profile was assessed by estimating shear stress by progressively deleting points from the top of the measured profile, while always retaining the lowest measurement. Second, the effect of variation in the minimum height of velocity measurement was explored by deleting points from the bottom of the profile, so that shear stress was calculated with progressively higher minimum depths, both throughout the full range of elevations of velocity measurement and with measurements obtained from only the bottom 20 per cent of the flow depth.

RESULTS

Velocity profiles for the three locations surveyed are shown in Figure 1. At the first site, the bed surface was smooth and the velocity measurements indicate a fully developed boundary layer with a log portion in the bottom 20 per cent of the flow (Figure 1a). The velocity profile from the second site also shows a logarithmic shape but velocities are still increasing at the highest point of measurement, indicating the presence of a depth-limited boundary layer (Figure 1b). The flow field is much more complex at the third location where bed roughness is considerably greater than at the two other sites. This is a special case where the channel was expanding and where most of the measuring points in the bottom 20 per cent of the flow were obtained from between roughness elements. The velocity profile clearly illustrates the presence of a skimming flow over more

Table I. Summary of field characteristics for the three field locations

Site	Water depth (m)	Bed material D_{50} (mm)	Bed material D_{84} (mm)	Minimum sampled elevation above bed (m)	Maximum sampled elevation above bed (m)	Number of average velocity measurements
1	0·225	0·39	3·25	0·0015	0·1480	20
2	0·150	1·93	8·88	0·0053	0·0719	16
3	0·150	21·41	36·76	0·0029	0·0640	21

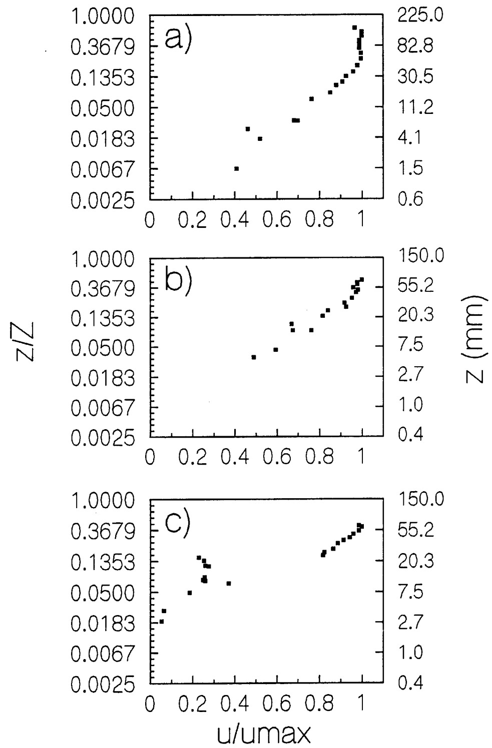

Figure 1. Plots of height above the bed (z) in non-dimensional (z/flow depth (Z)) and absolute values (log scale) against non-dimensional velocity (mean velocity at a point (u)/maximum mean velocity (u_{max})) for (a) Site 1, (b) Site 2 and (c) Site 3

slowly moving fluid close to the bed and from a zone where a high density of roughness elements resulted in greater local bottom shear. It gives rise to a marked zone of shearing at 24 mm (i.e. 16 per cent of the depth) above the bed, which approximately corresponds to the D_{50} of bed material.

Figure 2 shows the effect of varying the percentage of flow depth used to compute shear stress and roughness height values in the law of the wall. Roughness height is often used to check on the reliability of velocity profiles (Petit, 1990; Ferguson and Ashworth, 1992). For example, Ferguson and Ashworth (1992) considered values of z_0/D_{50} outside the range 0·1–0·5 as suspicious. However, the relationship between D_{50} and surface roughness may not be as strong as suggested by these authors because of hiding, packing and exposure effects. In Figure 2, all the roughness heights except a few points between 11 per cent and 16 per cent of flow depth at Site 3 (Figure 2c) fall within the 0·1–0·5 range. For the first two locations (Figure 2a,b), shear stress estimates increase with percentage elevation from the bed until they reach maximum values at heights around 20 per cent of the depth. This percentage of flow depth corresponds to the height of the logarithmic layer shown in Figure 1a,b. Using a progressively greater proportion of the flow above 20 per cent results in declining shear stress estimates. However, standard errors in shear stress are relatively large at Site 2 and the trend is not as clear as for Site 1. The pattern is the opposite for the site characterized by a zone of shearing between slow near-bed fluid and higher velocity fluid in the top portion of the flow (Figure 2c). In this case, shear stress decreases with a progressive

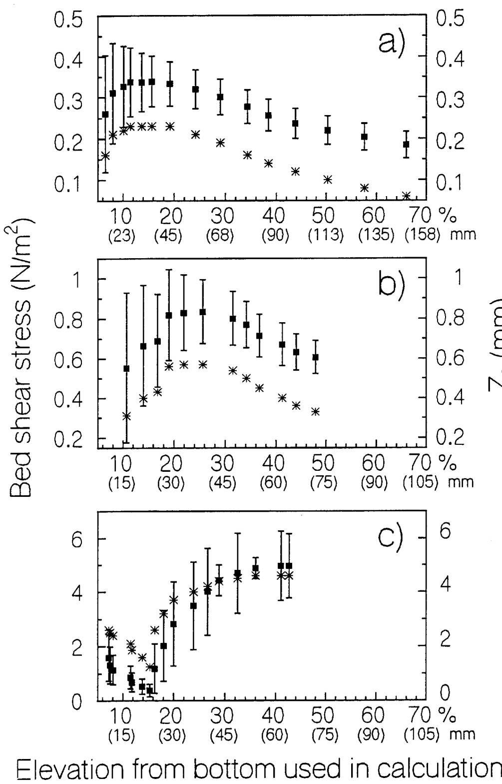

Figure 2. Variation in the estimation of bed shear stress (■) and roughness height z_0 (*) with the percentage of flow depth used in the computation at (a) Site 1, (b) Site 2 and (c) Site 3. Error bars correspond to one standard error in shear stress computed following Wilkinson (1984). Values of elevation in millimetres corresponding to the percentage of flow depth are also given in parentheses

increase in flow depth until it reaches an inflection point at about 16 per cent where shear stress increases rapidly up to about 35 per cent of flow depth where it becomes more or less constant. The inflection point corresponds to the shear layer height above the bed where velocities suddenly become much faster (Figure 1c).

Deviation from a log profile near the bed is expected in the presence of heterogeneous roughness elements because of form drag which reduces near-bed velocities (Wiberg and Smith, 1987). In such cases, the position of the velocity profile (e.g. over the top of a clast or in a hollow between clasts) is critical as the zero plane is highly variable. A displacement height can be used (Jackson, 1981; Robert, 1990; Ferguson and Ashworth, 1992) but this considerably alters the estimated boundary shear stress and creates the problem of finding a realistic value of displacement (Robert, 1990). Indeed, during transport-effective events, gravel-bed streams are often suspended-sediment laden (e.g. Ferguson and Ashworth, 1992; Ferguson *et al*., 1992) and knowing the D_{50} is not sufficient to assess the bed configuration and therefore to determine the proper displacement height.

The effect of varying the elevation of the first point of velocity measurement is illustrated in Figure 3 using the entire sampled flow depth to compute shear stress. Again, the pattern is similar for the two locations with finer bed material (Figure 3a,b) where shear stress decreases markedly as the elevation of the first measuring point increases. Roughness heights also decrease rapidly with measurement height and fall below the threshold of $0{\cdot}1D_{50}$ when the first measurement is higher than 4 mm above the bed for Site 1 (Figure 3a) and 16 mm for Site 2 (Figure 3b). The situation is more complex at the third location (Figure 3c). Shear stress increases until the first point is at about 17 mm (11 per cent) above the bed and then decreases abruptly to reach more or less constant values when measurement starts at 24 mm (16 per cent) and more above the bed. These constant values for roughness height fall below the $0{\cdot}1D_{50}$ (=2·14 mm) threshold.

When only the bottom 20 per cent of the flow is used to compute shear stress values, fewer points are used than in the previous set of results, so the standard error of shear stress is generally greater. For the first site, the

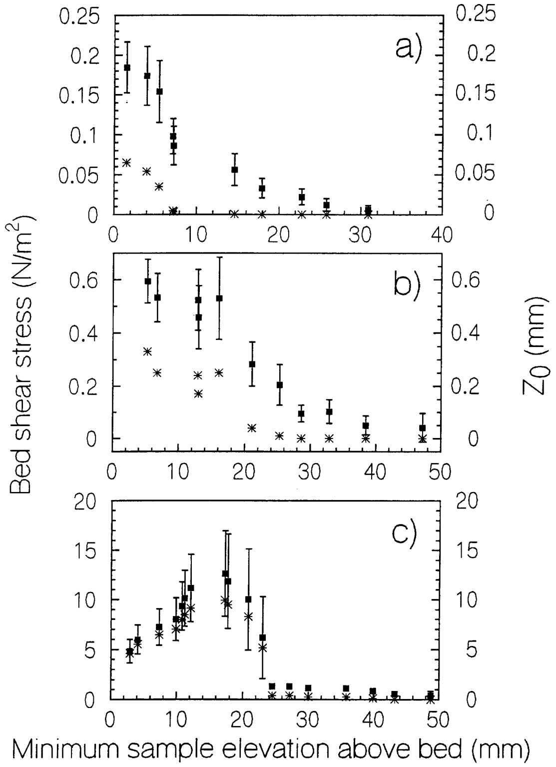

Figure 3. Variation in the estimation of bed shear stress (■) and roughness height (*) computed using the entire sampled flow depth as the minimal height above the bed increases at (a) site 1, (b) Site 2 and (c) Site 3. Error bars as in Figure 2

general trend shows a decrease in shear stress with higher minimum sampling elevations above the bed (Figure 4a), similar to Figure 3a but with greater scatter. Note that roughness heights computed using the three lowest minimum elevations above bed fall above the $0{\cdot}5\,D_{50}$ limit. However, the pattern is opposite for Site 2 (Figure 4b): shear stress increases with minimum sample elevation whilst it was decreasing when the full flow depth was used (Figure 3b). When the minimum sample elevation is greater than 13 mm, z_0 exceeds the $0{\cdot}5\,D_{50}$ value. As one would expect from Figures 1a and 1b, values of shear stress for the first two sites were always higher when computed using only the lower portion of flow, where velocity gradients were steeper. Finally, there is a considerable increase of shear stress estimate as the minimum elevation increases at the third location, with values up to 30 times higher when the minimal height is around 18 mm (12 per cent) (Figure 4c).

DISCUSSION

The results presented here clearly show that the range of measurement and alteration of the minimum sampling elevation both result in very different shear stress estimates. The absolute differences in bed shear stress for each subsampling scheme are necessarily site-specific but the conclusions are valid for any shallow gravel-bed river: the choice of measuring heights greatly affects the parameters obtained from the law of the wall. As these parameters (bed shear stress, shear velocity, roughness height) are used in bedload transport equations, it is important that the uncertainties related to the measurement positions are fully acknowledged.

High-resolution measurements are essential in order to identify the height of the logarithmic layer. It is not always possible to assume that it extends throughout the flow and hence to use a high proportion of flow depth to compute bed shear stress. Similarly, the logarithmic layer does not necessarily extend down to the bed when roughness elements are large (Wibert and Smith, 1987). This is the case at Site 3 where the lower limit of the logarithmic layer is at around 24 mm (i.e. $1{\cdot}12\,D_{50}$ or $0{\cdot}65\,D_{84}$) above the bed. It is clear in this case that shear

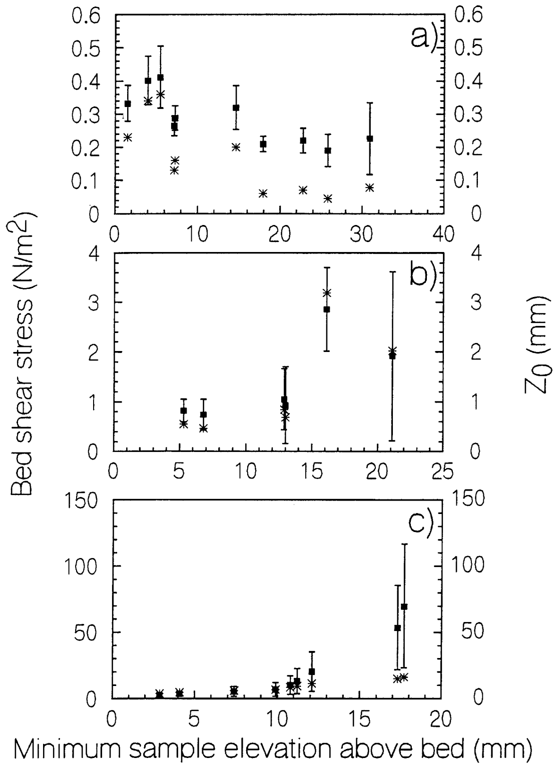

Figure 4. Variation in the estimation of bed shear stress (■) and roughness height (∗) computed using 20 per cent of the flow depth as the minimal height above the bed increases at (a) Site 1, (b) Site 2 and (c) Site 3. Error bars as in Figure 2

Table II. Bed shear stress and roughness height for different displacement heights at Site 3 (computed using velocity measurements above 24 mm)

Displacement height (mm)	τ_0 (Nm^{-2})	z_0 (mm)	z_0/D_{50}
0	1·34	0·39	0·02
10	2·10	1·25	0·06
20	3·01	2·76	0·13
24	3·42	3·56	0·17
30	4·07	4·96	0·23

stress can be greatly overestimated by the inclusion of near-bed points (e.g. left side of Figure 3c) or because the minimum sample elevation above the bed is too high when only 20 per cent of the flow depth is used (e.g. right side of Figure 4c). One alternative is to use only velocity measurements at heights greater than 24 mm to compute bed shear stress with the entire sampled flow depth. However, the roughness height value in this case is suspiciously low (z_0 = 0·39 mm = 0·02 D_{50}; Table II) and clearly a displacement height should be applied. In order to avoid applying a 'blind' value of displacement which greatly affects the bed shear stress estimate (cf. Robert, 1990), near-bed measurements are needed. As is shown in Table II, a sensible value of displacement appears to be around 24 mm, which corresponds to the knick point in the velocity profile. Hence, even if near-bed measurements are not used in the computation of bed shear stress, they remain essential in providing a good estimation of the height of bed roughness elements.

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